1. A large balloon is being filled with He from gas cylinders. The temperature is 25°C and the pressure is 1.00 atmosphere. The volume of the inflated balloon is 2000 m^3. What was the volume of He in the cylinders if the gas was under a pressure of 130 atmospheres and at a temperature of 12°C when in the gas cylinders?

   A) 32 m^3 B) 11 m^3 C) 34 m^3
   D) 15 m^3 E) 8 m^3

   solution:

   T, P, and V are (285 K , 130atm, V_i) initially, and are (298 K, 1.00atm and 2000 m^3) in the balloon. Since the number of moles doesn't change, $\frac{P_i V_i}{T_i}$ must be the same as $\frac{P_f V_f}{T_f}$. That makes $V_i = \frac{(1.00)(2000)(285)}{(298)(130)} = 15\text{m}^3$.

2. Which of the following statements about the ideal gas law, $PV = nRT$, is NOT true?

   A) When the volumes of two low density gases, which are initially at the same T, P and V, are compressed by the same fraction, their temperatures increase by that fraction.
   B) The ideal gas law is valid for high density of all gases.
   C) When the T of two low density gases, which are initially at the same T, P and V, is increased by the same amount, the V increases by the same amount.
   D) The value at $P = 0$ on a PV/nT vs P plot of any gas is the gas constant.
   E) All P-T plots for any gas at low density intercept the T-axis at the same point.

   solution:

   Ideal gases are 'rare' in the sense of rarified, or low density.

3. Boltzmann's constant, k, has a value of 1.381×10^{-23} J/K. What is the significance of the constant?

   A) $\mathcal{E}$ and $\mathcal{K}$
   B) $\mathcal{E}$: It defines a characteristic energy at the microscopic level given the temperature in Kelvins.
   C) $\mathcal{K}$: It measures the average kinetic energy per degree of freedom of molecules at any given temperature.
   D) $\mathcal{E}$ and $\mathcal{P}$
   E) $\mathcal{P}$: It allows the pressure of the gas to be calculated given the volume and temperature.

   solution:

   The average kinetic energy of a molecule in a gas at temperature T is $\frac{3}{2}kT$
   so the only wrong answer is that k gives P if V and T are known.

4. An ideal gas whose original temperature and volume are 37°C and 0.242 m^3 undergoes an isobaric expansion. If the final temperature is 76°C, then the final volume is approximately

   A) 0.170 m^3 B) 1.45 m^3 C) 0.0272 m^3 D) 0.0552 m^3 E) 0.272 m^3

   solution:

   Since $PV = nRT$ the ratio $\frac{PV}{T}$ doesn't change. If the gas starts at P_1, V_1, T_1 and finishes at P_2, V_2, T_2 then $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$. ($T$ in Kelvins, of course.) Isobaric means P doesn't change so $V_2 = \frac{T_2}{T_1} V_1 = 0.272\text{m}^3$.

5. A hailstorm causes an average pressure of 1.90 N/m^2 on the 190. m^2 flat roof of a house. The hailstones, each of mass 0.00800 kg, have an average velocity of 35.0 m/s perpendicular to the roof and rebound after hitting the roof with the same speed. How many hailstones hit the roof each second?

A) 2100 B) 650 C) 645 D) 1290 E) 1050

solution:

The force is the rate of change of momentum. The momentum change of one object is $2mv$ so the total momentum change per second, which is the force, is $2mvn$ where n is the number that hit and bounce back in one second. The pressure is the force per unit area so $n = \frac{PA}{2mv}$ which is 645 .

6. If the rms speed of oxygen molecules is 460 m/s at 0°C, the rms speed of oxygen molecules at 273°C is approximately

A) 1.84 km/s B) 230 m/s C) 920 m/s D) 651 m/s E) 325 m/s

solution:

$\frac{3}{2}kT = \frac{1}{2}mv_{\text{RMS}}^2$ so $\frac{v_{\text{RMS}}^2}{T}$ is the same for both T s. The ratio of the temperatures (in Kelvins!) is 2, so v_{273C} is $\sqrt{2}v_{0C} = 651$ m/s.

7. A lake with 6.50×10^9 kg of water, which has a specific heat of 4180 $\frac{\text{J}}{\text{kg C}^\circ}$, warms from 8.0 to 16.0°C. The amount of heat transferred to the lake is

A) 2.8×10^{16}J B) 2.5×10^3J C) 4.0×10^{15}J D) 1.7×10^{16}J E) 2.2×10^{14}J

solution:

The heat in is $mc\Delta T$ where m is the mass of the lake, c the specific heat of the water, and ΔT is the increase in temperature. That is $(6.50 \times 10^9)(8.0)(4180) = 2.2 \times 10^{14}$J.

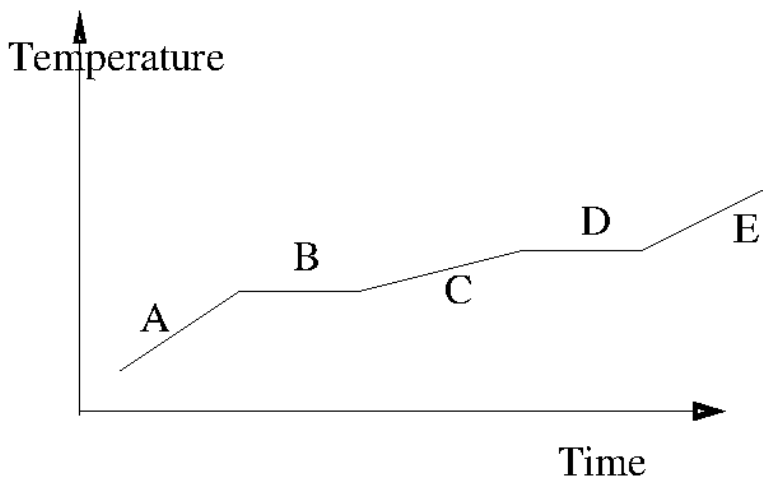

Figure 1: Problem 8

8. Heat is added to a substance at a constant rate. The substance starts as a solid and is melted; the liquid is heated and vaporized; finally, the vapor is heated. This process is shown in the graph. The specific heat of the solid can be found by

A) dividing the rate at which heat is added by the product of the slope of C and the mass of the substance.
B) multiplying the length of B (in seconds) by the rate at which heat is added, and dividing by the mass of the substance.
C) dividing the rate at which heat is added by the product of the slope of A and the mass of the substance.
D) multiplying the length of D (in seconds) by the rate at which heat is added, and dividing by the mass of the substance.
E) dividing the rate at which heat is added by the product of the slope of E and the mass of the substance.

solution:

Process A is the one in which the solid is being heated. The slope of the line is the temperature change per second. The specific heat is $\dfrac{\Delta Q}{m\Delta T} = \dfrac{\Delta Q/\Delta t}{m\Delta T/\Delta t}$. $\Delta Q/\Delta t$ is the rate of heat input and $\Delta T/\Delta t$ is the slope of the line. (T is the temperature, t the time and m the mass.)

9. The specific heat of a gas is

A) greater at constant pressure than at constant volume.

B) directly proportional to the absolute temperature.
C) the same for all gases.
D) independent of constraints imposed on it while heating.
E) a negligible quantity.

solution:

Heating a gas at constant volume is done without doing work. Heating it at constant pressure allows the gas to expand and thus do work. That work requires that more heat be supplied, so the specific heat is greater.

10. Besides Joule's classic experiment, another way of demonstrating the equivalence between mechanical energy and heat is the following: Put some lead shot into a glass tube, seal both ends of the tube, invert the tube quickly several times, and measure the temperature of the shot. If you assume that all the mechanical energy is converted into heat in the lead shot and that no heat is lost, what is the change in the temperature of the shot if the tube is 1.5 m long, there are 0.15 kg of shot, and the tube is inverted 20 times? (The specific heat of lead is 128. J/kg °C.)

A) 7.7 °C B) 0.077 °C C) 0.25 °C D) 2.3 °C
E) 2.5 °C

solution:

The potential energy of the lead is converted into kinetic energy which is converted into heat. The amount per inversion is mgh where $m = 0.15$ kg, $g = 9.81$ m/s and $h = 1.5$ m. During 20 inversions the heat in is therefore $20mgh$. Heat in $= mc\Delta T$ so $\Delta T = \dfrac{44.}{19.} = 2.30$°C.

11. An ideal gas changes, reversibly and quasistatically, from state i to state f along paths iaf and ibf. If the heat added along iaf is 50 cal then the work done by the gas along iaf is 20 cal. If the heat added along ibf is 40 cal, what must the work done along ibf be?

A) 20 cal B) 50 cal C) 30 cal D) 40 cal E) 10 cal

solution:

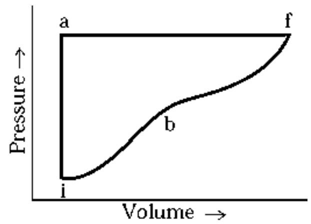

Figure 2: Problem 11

The first law tells us that the heat in is the work done by the substance plus the increase in its internal energy. The internal energy is a state variable, that is something that depends only on the state of the substance, and is not determined by how the substance came to be in that state. Therefore the difference between the heat in and the work done is the same along *any* path from i to f as it is along any other.

That difference is 30 cal along iaf so is also 30 cal along ibf. $40 - 30$ is 10.

12. The equation of state for a certain gas under isothermal conditions is $PV = 38.4$ where the units are SI. The work done by this gas as its volume increases isothermally from $V_i = 0.200\ \mathrm{m}^3$ to $V_f = 0.800\ \mathrm{m}^3$ is approximately

    A) 115 J B) 43.3 J C) 53.2 J D) 28.6 J E) 2.86 J

    solution:

    The work done during an isothermal process is $\int P\,dV = \int \frac{nRT}{V} dV = nRT \ln \frac{V_\mathrm{f}}{V_\mathrm{i}}$. Since $PV = nRT$ this is $38.4 \ln \frac{V_\mathrm{f}}{V_\mathrm{i}}$ which is 53.2 J.

13. The internal energy of a solid depends on the number of degrees of freedom available to each atom. Which of the following statements is correct concerning these degrees of freedom in a solid?

    A) Three are due to translational, and two are rotational.
    B) Three are vibrational, and three are rotational.
    C) Three are due to kinetic energy and three from vibrational potential energy.
    D) Three are due to translational, two are rotational, and one is vibrational.
    E) There are three, and all are due to translational motion.

    solution:

    Each atom has the same 3 degrees of translational motion that any object in 3-space has. Each is bound to its position in the solid by a restoring force in each of the three orthogonal directions. Thus there is potential energy associated with each of those directions. That makes six in all.

14. An ideal gas with an initial volume of 4.00 L at a pressure of 4.00 atm is compressed adiabatically until it has a volume of 1.00 L and then cooled at constant volume until its temperature drops to its initial value. The final pressure is

    A) 2.40 atm B) 0.75 atm C) 16.0 atm D) 32.0 atm E) 1.33 atm

    solution:

	P	V	T
A	4.00	4.00	T_0
B	P_B	1.00	T_B
C	P_C	1.00	T_0

Call state A: $(P, V) = (4.00, 4.00, T_0)$, B: $(P_B, 1.00, T_B)$ and C: $(P_C, 1.00, T_0)$. Use the ideal gas law for C and A : $P_C \times 1.00 = nRT_0$, $4.00 \times 4.00 = nRT_0$. Equate the two: $P_C \times 1.00 = 4.00 \times 4.00$. That is $P_C = 16.0$ atm.

The nature of the processes that bring the gas to the various states need not be used; it would be needed if T_B or P_B were to be found.