(If you have any questions about these solutions please email me or drop in and ask)

Sept 13 answers

1. An object, mass 2.0 kg, velocity 20 m/sec in the direction perpendicular to a wall, hits that wall and bounces directly back, elastically. Find its momentum change. Don't forget that momentum is a vector.

Answer: Suppose it's going towards the right. Its momentum is (2.0 kg)(20 m/sec)=40 kg m/sec, to the right. 'Elastic' means its energy doesn't change. That implies that its speed doesn't change. So its momentum after the bounce is 40 kg m/sec to the left. That means a momentum vector of 80 kg m/sec to the left must have been added to its initial momentum.

2. What is the average kinetic energy of all the molecules in an ideal gas in a container if the temperature of the gas is 267 K? What further information is required to determine the total kinetic energy of the molecules?

$R = 8.3 \frac{\text{J}}{\text{mole K}}$; $k = 1.4 \times 10^{-23} \frac{\text{J}}{\text{K}}$.

Answer: Suppose it's monoatomic. Then the average KE of a molecule is $3kT/2$ where T is 267 K.

The total is the average multiplied by the number of molecules in the container.

3. The Maxwell-Boltzmann distribution of molecular speeds is

$$Cv^2 e^{-\frac{mv^2}{2kT}}.$$

Find that speed which more molecules have than any other speed. Give your answer in terms of $m, v, k,$ and T; they have their usual meanings; C's value is irrelevant to the question.

Answer: That speed is the one which makes the function a maximum. The maximum is found by taking the derivative of $v^2 \exp(-\frac{mv^2}{2kT})$, setting it equal to zero and solving for v. The result is $v = \sqrt{2kT/m}$.

4. These are the values of the pressure of a fixed volume of of a fixed mass of Nitrogen gas as a function of Celsius temperature.

Plot these data on the graph paper below to estimate the value of absolute zero on the Celsius scale. Use Celsius temperature on the $x-$axis.

T (°C)	-50	0	50	100	150	200
P (10^4 N/m^2)	8.0	10.0	11.5	13.0	15.0	16.5

Answer: Plot T on the horizontal axis and P on the vertical. Make sure the T-values go as low as -300. Plot the points and extrapolate the line along which they lie until that line crosses the T-axis. The value of T at that point is your estimate of absolute zero.

5. 1.4 kilograms of an ideal gas whose molecular weight is 32 is in a container of volume 2.5m^3 at a pressure of 3.0×10^5N/m^2. Find its temperature.

Answer: Use the ideal gas law $PV = nRT$. $T = \frac{PV}{nR}$. n is the number of moles. That is the mass divided by the molecular weight. That makes T 21 K.

6. Pretend that the molecules in a closed rectangular container whose volume is 0.150 m^2 all have the same speed, 5.4 m/sec, and 1/3 of them are traveling in straight lines between each of the three pairs of parallel sides, perpendicular to those sides, half going one way and the other half going the opposite way. Suppose that there are 3.5×10^{20} molecules in all. Determine how many hit an area of 0.10 m^2 each second. Pretend there is no gravity and that the molecules exert no force on each other.

Answer: The number that hit in a second is the number that are within 5.4 meters of the wall and heading towards it. The number that hit 0.10m^3 is (1/6) of number within a volume of (0.10)(5.4)=0.54 m^3.
The number in that volume is the number per unit volume times 0.54 m^3.
The answer is $(1/6)(.54)(3.5\times10^{20}/0.150)=2.1 \times 10^{20}$.

7. 1.40 kilograms of an ideal gas whose molecular weight is 32 is in a container of volume 2.10m^3 at a temperature of 457 K. Find its pressure.
The universal gas constant R is 8.314 $\dfrac{\mathrm{J}}{\mathrm{mole\,K}}$.

Answer: Use the ideal gas law $PV = nRT$. $T = \dfrac{PV}{nR}$. n is the number of moles. That is the mass divided by the molecular weight. That makes T 21 K.

8. Find the pressure, in Newtons per square meter, of 3.0 moles of an ideal gas whose volume is 0.030 meter^3 and whose temperature is 300°A.

Avagadro's number is 6.0×10^{23}molecules per mole.
Boltzmann's constant=$1.38 \times 10^{-23}\mathrm{m}^2\ \mathrm{kg}\ \mathrm{sec}^{-2}\ \mathrm{K}^{-1}$.

answer: This requires a simple direct application of the ideal gas law.
$$P = \frac{nRT}{V} = \frac{3.0 \times 8.31 \times 300}{.030} = 2.5 \times 10^5 \mathrm{N/m^2}.$$

9. On a day when the air temperature is 25 °C the pressure in an auto tire is 32 psi. What would the tire gauge read on the next day, when the atmospheric pressure is the same but the air (and tire) temperature is 15 °C?

answer: (suppose the volume doesn't change; suppose atomospheric pressure is 14.7 psi)
Day 1: T=298 K, P=(32+14.7)psi
Day 2: T=288 K, P=?
PV=nRT; (P/T)=(nR/V) so P/T on day 1 is the same as P/T on day 2.
P, day 2 = (T, day 2) times (P/T), day 1
$P_2 = T_2 \times P_1/T_1 = 288 \times 46.7/298 = 45.1$ psi, absolute.
$P_{2,\mathrm{gauge}} = P_{2,\mathrm{absolute}} - P_{\mathrm{atmospheric}} = 45.1 - 14.7 = 30.4$ psi.
Notice the use of psi here is perfectly ok.

11. A pail contains 2.00 kg of water at 310 K. 50 grams of ice, at 273 K, is placed into it. When the mixture comes to thermal equilibrium what is its temperature? Pretend that the pail has zero heat capacity and no energy is lost to or gained from the surroundings.

answer:

Suppose all the ice melts. Suppose the final temperature is T_f. The total heat into the ice

$m_{\mathrm{ice}}L_{\mathrm{ice}} + (T_f - 273)c_{\mathrm{liquid}}m_{\mathrm{ice}}$

The total heat out of the water is $m_{\text{water}} c_{\text{liquid}} (310 - T_f)$

Equate the two and solve for T_f. The result is 307 K.

Sept 20 Answers

1. The internal energy of a solid depends on the number of degrees of freedom of its constituent degrees of freedom. Are they associated with the kinetic energy of the molecules or the potential energy of the molecules? How many are there of each type? Imagine that number to be 9. It's not, in fact. But if it were, what would the molar heat capacity of the solid be, according to the law of equipartition of energy. Why?

Answer: There are three associated with KE and three with PE.

If there were 9 d.o.f. the total energy per mole would be $\frac{9RT}{2}$.

The energy at $T+1$ would be $\frac{9R(T+1)}{2}$ so the energy needed to bring it from T to $T+1$ is the difference beween the two, which is $\frac{9R}{2}$.

That is (would be) the heat capacity of a mole.

2.a) How many degrees of freedom does a monoatomic molecule of an ideal gas have?

Answer: Three.

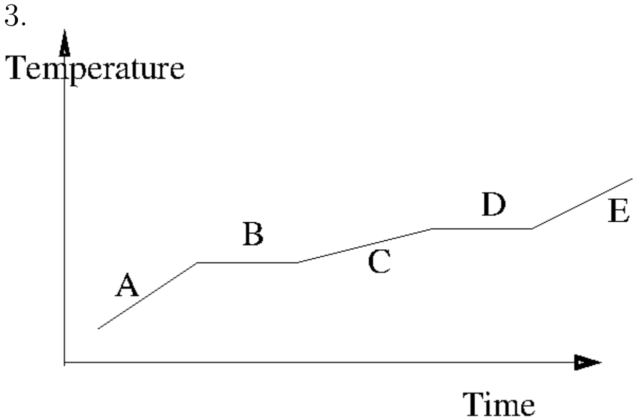
b) According to the principle of equipartition of energy, what is the average energy of the molecules in a gas made of such molecules?

Answer: $\frac{3kT}{2}$.

c) Use the answer to b) to calculate the molar heat capacity of an ideal gas. Show your work.

Answer: $\frac{3kN_A(T+1)}{2} - \frac{3kN_AT}{2}$ which is $\frac{3R}{2}$.

3.



Heat is added to a mass m of substance at a (known) constant rate. The substance starts as a solid and is melted; the liquid is heated and vaporized; finally, the vapor is heated. This process is shown in the graph.

Use the graph to find the heat of fusion of the solid and the heat of vaporization of the liquid.

Use the value 12 Joules/sec for the rate and 25 grams for m.

	begin A	begin B	begin C	begin D	beginE	end E
time(sec)	50.0	100.0	110.0	160.0	180	220
Temp ($^\circ C$)	10	50	50	90	90	100

answer:

The leftmost part of the graph describes the solid, as it warms. The next section describes melting. The third, the second sloping part, is the warming of the liquid. The following horizontal section is the boiling, which like the melting, happens at constant temperature. Finally the last, sloping, section, is the heating of the vapor phase.

some papers asked for the following three things:

a)The specific heat of the solid is found using the leftmost sloping section. The heat input during that section is the rate times the duration. The temperature increase is the difference between the T value at the right-hand (upper) end of that sloping section and the T value at its lower (left-hand) end. The specific heat is the heat input divided by (temperature change times mass).

b) The specific heat of the liquid is the rate of heat input times the duration of the middle sloping line divided by the (temperature increase during that section times the mass).

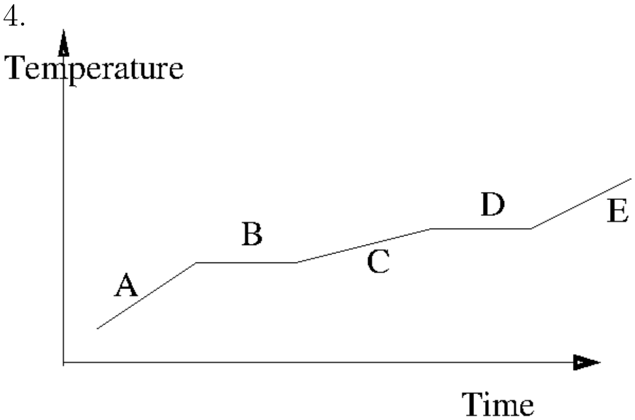
c) The specific heat of the vapor is the rate of heat input times the duration of the far-right sloping line, divided by (the mass times the temperature increase during that section).

others asked for these two things:

d) The latent heat of melting ('fusion') is the rate of heat input times the duration of the first constant-temperature section, divided by the mass.

e) The latent heat of vaporization is the rate of heat input times the duration of the second constant-temperature section, divided by the mass.

4.



Heat is added to a mass m of substance at a (known) constant rate. The substance starts as a solid and is melted; the liquid is heated and vaporized; finally, the vapor is heated. This process is shown in the graph.
Use the graph to find the specific heat of the solid, of the liquid and of the vapor.
Use the value 10 Joules/sec for the rate and 20 grams for m and find those specific heats.

	begin A	begin B	begin C	begin D	beginE	end E
time(sec)	50.0	100.0	110.0	160.0	180	220
Temp (°C)	10	50	50	90	90	100

Answer: See answer to previous question.

5. You want to heat a cup of water by dropping a 100 gm pebble into it, over and over again. Say you drop it from 1 meter above the cup. How many times would you have to drop it in order to increase the temperature by 1°C? Suppose the cup holds 0.3 kg of water. Neglect the heat need to warm the cup or the pebble and pretend that the pebble can be removed with no water attached.

The heat capacity of water is 4.18kJoules/(kg °C).

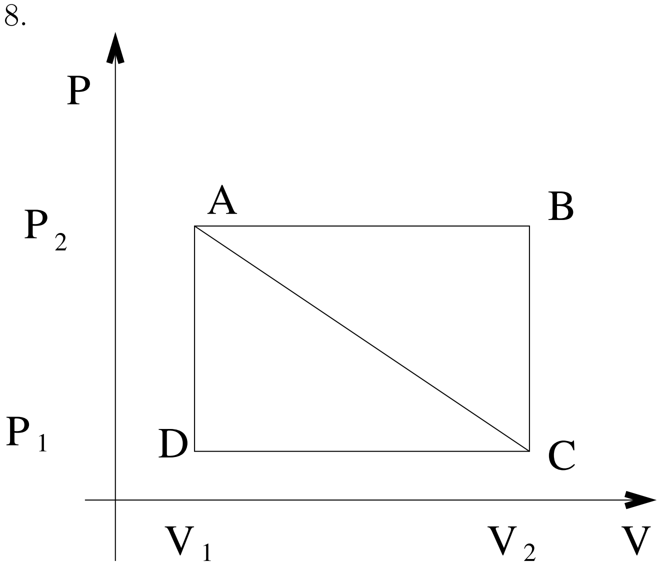
Answer: Call the length of the fall H,the pebble's mass m and $c_W, m_W and \Delta T_W$ the specific heat, mass and temperature increase of the water, respectively.
The amount of KE dissipated as heat for a single pebble drop is the amount of PE which has changed to KE in the fall. That is mgH=0.98 Joules.
The heat needed is $c_W m_W \Delta T_W$. That is (4.18)(0.3)(1)=1.25 Joules.
You need drop it only twice.

6. Suppose the RMS speed of the (monoatomic) molecules comprising a gas is 540 m/s at $T = 400$K. Find the RMS speed of the molecules if the temperature is 450K.

Answer: The average energy of a molecule is $\frac{3kT}{2}$. That is all KE, $\frac{mv^2}{2}$. The square root of $\frac{2}{m}$times that KE is the definition of RMS speed.
$V_{400} = \sqrt{\frac{3k400}{m}} = 540$ and $V_{450} = \sqrt{\frac{3k450}{m}}$.
That gives $V_{450} = \sqrt{\frac{450}{400}} \times 540 = 572K$.

7. A gas absorbs 3.5×10^6 J and does 450×10^3 J of work while doing that. By how much does its internal energy change?

Answer: The First Law is heat into something = work done by the thing + increase in internal energy of the thing, $đQ = dU + PdV$ for short.
You are given $đQ$ and dW. dU is $3.5 \times 10^6 - 450 \times 10^3 = 3050 \times 10^3$Joules.
8.



Find the work done by an ideal gas when it goes through the processes

1) A –> B; 2) A –> C; 3) A –> B –> C –> D –>A .

Answer:

A->B: $P_2(V_2 - V_1)$
A->C: $P_1(V_2 - V_1) + (1/2)(P_2 - P_1)(V_2 - V_1)$.
A->B: $P_2(V_2 - V_1)$.
D->A: Zero.
The fact that it's an ideal gas is irrelevant. The work is the area under the line segment,

9. What does the equipartition theorem predict for the specific heat, in calories per gram, of solid iron? What is the measured specific heat of solid iron? Repeat for solid aluminum. (some papers: copper instead of aluminum)

answer:

The law of Dulong and Petit, which follows from the equipartition theorem, says that the heat capacity per mole, of a solid, is $3R$. The heat capacity per gram is the heat capacity per mole divided by the number of grams in a mole, aka the molecular weight (more precisely the molecular mass). That is $3R$/(molecular weight).
So the problem requires the molecular weights of iron and aluminum, which are 55.8 and 27.0, respectively. The gas constant R is 1.987 calories/(mol-K) so the predictions are 0.107 and .221.
These are in very good agreement with the measured values.

Sept 27 Answers

1. Three moles of an ideal gas are transformed among the following states:

	P(atm)	$V(10^{-3}\text{m}^3)$
A	4.0	4.01
B	4.0	
C	1.0	20.0
D	1.0	
E	1.0	4.01

a) Draw these points on a labeled P, V diagram.

Answer: Only A, C and E can be located since they are the only points with both P and V specified. B is somewhere along P=4.0 and D is somewhere along P=1.

b) If A is brought isobarically to B and then isothermally to C, find T_B , V_B and T_C.

Answer: If the path from B to C is isothermal then T_B must be the same as T_C and since P_C and V_C are both given, $T_C = \frac{P_C V_C}{nR}$ and T_B is the same. Use $R = 8.2 \times 10^{-2}$ liter-atm/(mol K) and the arithmetic is simple. Then use P_B and T_B to find V_B.

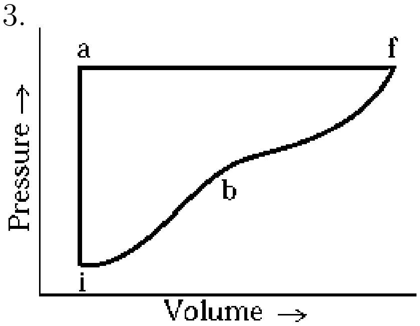
c) Find the work done by the gas on its way from A to C, and the heat it absorbs, and its change in internal energy.

Answer: The work done from A to B is (4.0)(20.00-4.01) liter-atm. From B to C it is $\int_{V_B}^{V_C} \frac{nRT}{V} dV = nRT_C \ln(V_C/V_B)$. The heat needed to bring it from A to B is $C_P(T_B - T_A)$. Because (heat into) = (work done by) plus (increase in internal energy) and for an ideal gas internal energy depends only on temperature then, from B to C, heat in equals work done by the gas. That work has just been found. Finally the internal energy change all takes place along A to B and it is $\frac{3nRT_B}{2} - \frac{3nRT_A}{2}$.

2. An ideal gas with an initial volume of 5.6 liters at a pressure of 2.4 atm is compressed adiabatically until it has a volume of 3.7 liters and then cooled at constant volume until its temperature drops to its initial value. Find its final pressure.

Answer: Call the start 'A' and the finish 'C'. $T_A = \frac{P_A V_A}{nR}$. The final temperature is the same. The final pressur obeys $P_C V_C = nRT_C$. That is $P_C = \frac{nR}{V_C} T_C = nR \frac{P_A V_A}{nR} \frac{1}{V_C} = \frac{P_A V_A}{V_C}$
That is (5.6)(2.4)/3.7=3.6 atm.
The fact that the first part of the process is adiabatic is not needed. Any path that compresses the gas to 3.7 liters and then cooled at constant V to its initial temperture would give the same final V and T and therefore the same P.

3.



An ideal gas changes, reversibly and quasistatically, from state i to state f along paths iaf and ibf. If the heat added along iaf is 100 cal then the work done by the gas along iaf is 60 cal. If the heat added along ibf is 70 cal, then what must the work along ibf be?

Answer:

> The key point here is that the change in internal energy is the same along both paths, because it depends only on the state, not the process.
>
> Along iaf, since heat in is 100 cal and work done is 60 cal, the internal enrgy increases by 40 cal. If the heat in along ibf is 70 cal, since the internal energy increase must be 40 cal the work done must be 70-40=30 cal.

4. The equation of state for a certain gas under isothermal conditions is $PV = 45.2$ where the units are SI. Find the work done by this gas as its volume increases isothermally from $V_i = 2.1$ m^3 to $V_f = 4.2$ m^3.

Answer:

> The work done during an isothermal process is
> $\int PdV = \int_{V_i}^{V_f} \frac{nRT}{V} dV = nRT \ln \frac{V_f}{V_i} = nRT \ln \frac{4.2}{2.1}$.
>
> Since $PV = nRT$, $nRT = 45.2$, and the answer is $45.2 \ln \frac{4.2}{2.1} = 31$ Joules.

5. Three moles of a monoatomic ideal gas are transformed among the following states:

	P(atm)	$V(10^{-3}\text{m}^3)$
A	4.0	4.01
B	4.0	
C	1.0	20.0
D	1.0	
E	1.0	4.01

a) Draw these points on a labeled P, V diagram.

Answer: A, C and E can be located precisely. D lies somewher along the line P=1 and B along P=4.

b) If A is brought isothermally to D and then isobarically to E, find T_D and V_D.

Answer: That means $T_A = T_D$ and $P_D = P_E$. P_D and V_D give you T_D, and T_D and P_D give you V_D.

c) Find the work done by the gas on its way from A to E, the heat it absorbs and its change in internal energy.
Answer: (See the answer to problem 1, Sept 27,above. The method there will work here with only slight modification.)

6. 2.0 moles of a diatomic ideal gas are transformed from A to B and back to A. From A to B the path is a straight line on the PV diagram; from B back to A the process is adiabatic.

	P(atm)	$V(10^{-3}\text{m}^3)$
A	2.1	10.
B	1.1	21.

a) Draw these points on a labeled P, V diagram.

Answer: You know how.

b) Find the change in internal energy of the gas during the cycle.

Answer: The internal energy depends only on the state or the condition of the substance. If it's back to the same state the internal energy is back to the same value. Its change is zero.

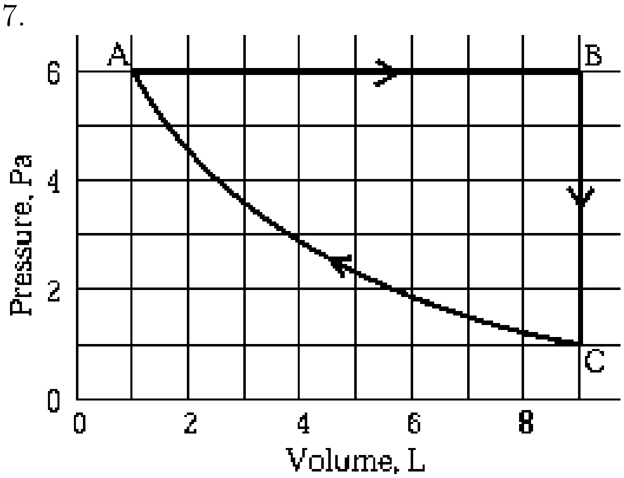
c) Find the work done by the gas during the cycle.

Answer: Work from A to B is area under the straight line from A to B. That is 1.1 * (21-10) +(1/2)(2.1-1.1)*(21-10)=13.6 $\times 10^{-3}$ atm m^3. On the way back, work is done on the gas. Since there is no heat into or out of the gas along the adiabate the First Law says the work done on the gas is its increase in internal energy. The internal energy at A is $(5/2)nRT_A$ and at B is $(5/2)nRT_B$. The increase is $(5/2)(nR)(23.1 - 21.0)/(nR)$=5.3 $\times 10^{-3}$atm m^3. The net amount is $13.6 - 5.3 = 8.3 \times 10^{-3}$ atm m^3.

d) Find the heat absorbed by the gas during the cycle.

Answer: The heat absorbed must be the same as the work done by the gas since the internal energy is the same at the end as it was art the beginning. That is 8.3×10^{-3} atm m^3.

7.



A substance is taken through the cycle shown. The processes are quasistatic. Find the work done by the substance during one cycle from A to B to C and back to A.

Answer: The easiest way to do this problem is to find the area inside the line that represents the cycle by counting the number of boxes inside and multiplying that by the area of one box. One box has area 1 liter times 1 Pascal. I count 21 full boxes and about 5 more from the 10 that lie partially within the line. My estimate for the answer is thus 26 liter Pascals.

8. 2 moles of an ideal gas undergoes an isothermal expansion from ($P_0 = 1.2 \times 10^5$ Nt/m^2,$V_0 = 0.2$m^2) to $V_1 = 0.4$m^2. Find the work the gas does during the expansion. Start from $dW = PdV$.

Answer: Use $PV = nRT$ and integrate. That gives work= $\int_{V_0}^{V_1} \frac{nRT}{V} dV$. That is $nRT \ln \frac{V_1}{V_0}$.

T is $\frac{P_0 V_0}{nR}$ or $\frac{P_1 V_1}{nR}$.

Work is $nR \times \frac{P_0 V_0}{nR} \times \ln \frac{V_1}{V_0} = 1.2 \times 10^5 \times 0.4 \times \ln \frac{0.4}{0.2} = 0.33 \times 10^5$ Joules