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Supplementary problem set I (Fall, 2017)

Problems

1. Consider real, diagonal matrices in d dimensions. Under what conditions are these matrices orthogonal? How many such proper rotation matrices are there and how many improper ones?

2. Consider n-component vectors with complex elements and define an inner-product by

$$(\mathbf{a}, \mathbf{b}) = \sum_{i=1}^{n} a_i^* b_i. \tag{1}$$

   (a) Verify that Eq. (1) satisfies the properties of an inner product.

   (b) Derive what property a matrix $\boldsymbol{R}$ with complex elements should satisfy so that

$$(\mathbf{Ra}, \mathbf{Rb}) = (\mathbf{a}, \mathbf{b}). \tag{2}$$

3. Similarly, consider complex-valued, square-integrable functions of one variable, and define an inner-product by

$$(f, g) = \int f(x)^* g(x)\, dx.$$

   Answer the same questions as in the previous problem.

   Hint: The analog of δ_{ij}, the Kronecker-δ, for continuous variables is the Dirac-δ-function. The analog of summation over a discrete variable is integration. For example, multiplication of a vector by a matrix becomes integration as follows:

$$b_i = \sum_j T_{ij} a_j \to g(x) = \int T(x, x') f(x')\, dx'.$$

   T is usually called an integral kernel, but you can think of it as a matrix with continuous indices. Clearly, $T(x, x') = \delta(x - x')$ plays the role of the identity in this continous matrix multiplication.