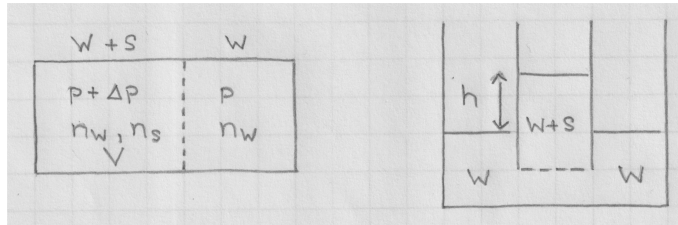


Osmotic pressure [tln26]

Consider a dilute solution. It consists of a solvent (e.g. water) and a solute (e.g. sugar). System A (pure solvent) is separated from system B (solution) by a membrane that is permeable to the solvent only.

At thermal equilibrium, this causes an excess pressure in system B , which is named *osmotic pressure*.

The number density n_S of solvent in system B is assumed to be much smaller than the number and n_W of solute: $n_S \ll n_W$ (dilute solution).



Condition of chemical equilibrium for solute across membrane:

$$\mu_W^{(A)} = \mu_W^{(B)} \Rightarrow \Delta\mu_W = 0.$$

The solute chemical potential is affected in two ways:

- (1) Effect of change in solvent concentration: Use the result of [tex173].

$$\Delta\mu_W^{(1)} = RT \ln \left(\frac{n_W}{n_W + n_S} \right) = RT \ln \left(\frac{n - n_S}{n} \right) \simeq -RT \frac{n_S}{n}.$$

- (2) Effect of change in pressure:

Use a Maxwell relation inferred from $dG = -SdT + Vdp + \mu dn$:

$$\left(\frac{\partial\mu_W}{\partial p} \right)_{T, n_W} = \left(\frac{\partial V}{\partial n_W} \right)_{T, p} \simeq \frac{V}{n} \Rightarrow \Delta\mu_W^{(2)} = \frac{V}{n} \Delta p.$$

At thermal equilibrium: $\Delta\mu_W^{(1)} + \Delta\mu_W^{(2)} = 0 \Rightarrow \frac{V\Delta p}{n} - RT \frac{n_S}{n} = 0.$

Van't Hoff's law for osmotic pressure: $\Delta p \doteq \pi = \frac{RTn_S}{V}.$

Application shown: $\pi = \rho gh$, where ρ is the mass density of the solution.