Superconducting transition [tln35]

Perfect conductor versus superconductor:

The (hypothetical) *perfect conductor* and the (real) *superconductor* are materials that support steady currents without any voltage source driving them.

Relation between electric field $\mathbf{E}$ and current density $\mathbf{J}$ in normal conductor:

$$\mathbf{E} = \rho \mathbf{J}.$$

In a perfect conductor, the resistivity vanishes below a critical temperature: $\rho(T) = 0$ at $T < T_c$, implying $\mathbf{E} \equiv 0$ inside.

According to Faraday's law, $\nabla\mathbf{E} = -\partial\mathbf{B}/\partial t$, an identically vanishing electric field $\mathbf{E}$ freezes the magnetic field $\mathbf{B}$ in the same region.

- If the perfect conductor is cooled below T_c at zero external magnetic field and then an external magnetic field turned on, it cannot penetrate.
- If the perfect conductor is cooled below T_c in an external magnetic field, which is then turned off, the field will stay nonzero inside.

The attribute "zero resistivity" of a perfect conductor does not describe a thermodynamic state. The state depends on how it is arrived at.

In a superconductor the permeability vanishes below a critical temperature: $\mu = 0$ at $T < T_c$, implying $\mathbf{B} \equiv 0$ inside.

The primary attribute of a superconductor is that it is a perfect diamagnet. The attribute "zero resistivity" is secondary.

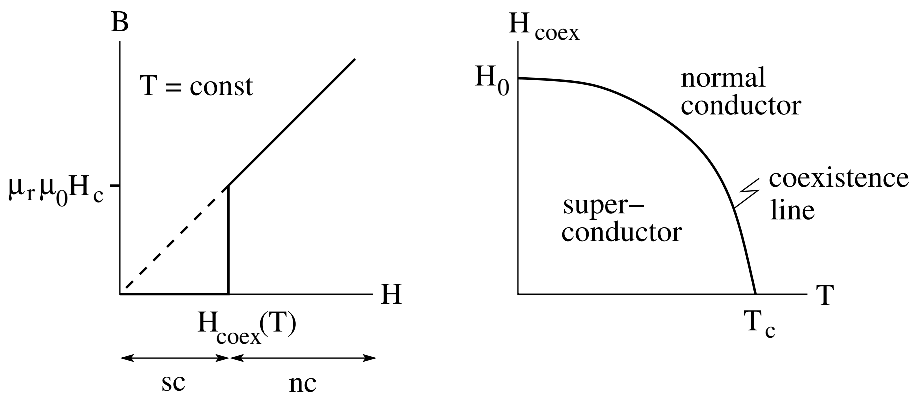
- If the superconductor is cooled below T_c at zero external magnetic field and then an external magnetic field turned on, it cannot penetrate.
- If the superconductor is cooled below T_c in an external magnetic field, the magnetic field will be expelled.

The attribute "zero permeability" of a superconductor does describe a thermodynamic state. The state is independent of how it is arrived at.

Meissner-Ochsenfeld effect:

Thermodynamics of a type-I superconductor:

- The magnetic induction $B = \mu_r \mu_0 H$ is expelled by surface supercurrents from the interior of the superconductor for external magnetic fields $H < H_{\rm coex}(T)$.
- The function $H < H_{\rm coex}(T)$ is monotonically decreasing with T and vanishes at $T_{\rm c}$, implying that a sufficiently strong external magnetic field destroys superconductivity at any temperature.



Coexistence condition between the superconducting phase and the normal conducting phase:[1]

$$G^{(sc)}(T,H) = G^{(nc)}(T,H).$$

Change of Gibbs free energy along the coexistence line:

$$dG^{(sc)} = dG^{(nc)} \quad \Rightarrow \quad -S^{(nc)}dT - B^{(nc)}dH = -S^{(sc)}dT - B^{(sc)}dH$$

with $B^{(nc)} = \mu_r\mu_0 H_{coex}(T)$ and $B^{(sc)} = 0$.

The term $B^{(nc)}dH$ represent an increment of magnetic-field energy inside the normal conductor.

Clausius-Clapeyron equation adapted to this situation:

$$S^{(nc)} - S^{(sc)} = -\mu_r\mu_0 H_{coex}(T)\left(\frac{dH}{dT}\right)_{coex}.$$

Latent heat: $L = T\left(S^{(nc)} - S^{(sc)}\right)$.

[1]Alle extensive quantities in this application are per unit volume.

As H increases, $G^{(sc)}$ stays constant but $G^{(nc)}$ decreases:

$$G^{(nc)}(T,H) - G^{(nc)}(T,0) = -\int_0^H B^{(nc)} dH = -\frac{1}{2}\mu_r\mu_0 H^2.$$

On the coexistence line: $G^{(nc)}(T,H_{coex}) = G^{(sc)}(T,H_{coex})$.

$$\Rightarrow\; G^{(sc)}(T,0) - G^{(nc)}(T,0) = -\frac{1}{2}\mu_r\mu_0 H_{coex}^2(T).$$

Additional empirical information is required for the derivation of more specific results, such as the latent heat and the heat capacity [tex44].

Example: empirical formula for the coexistence line:

$$H_{\rm coex}(T) = H_0\left(1 - \frac{T^2}{T_{\rm c}^2}\right), \quad 0 \le T \le T_{\rm c}.$$