

[tex10] Work extracted from finite heat reservoir in finite environment

A (finite) heat reservoir with heat capacity $C_H = \text{const}$ is initially at temperature T_H and the (finite) environment with heat capacity C_L at the lower temperature T_L .

(a) When heat is allowed to flow from the reservoir to the environment, both will end up at the temperature,

$$T_f = \frac{C_H T_H + C_L T_L}{C_H + C_L} \quad (\text{arithmetic mean}).$$

Verify this and determine the total amount of heat ΔQ that has been transferred.

(b) When the reservoir is connected to the environment by a Carnot engine which absorbs an infinitesimal amount of heat δQ per cycle, converts part of it into work δW , and dumps the rest into the environment, the final common temperature of the reservoir and the environment will be

$$T_f = T_H^{C_H/(C_H+C_L)} T_L^{C_L/(C_H+C_L)} \quad (\text{geometric mean}).$$

Verify this and determine the total amount of work ΔW that has been extracted from the system. The fraction of the excess internal energy $U_{ex} = C_H(T_H - T_L)$ that can be converted into work is characterized by the quantity $\Delta W/U_{ex}$. Plot this quantity versus the reduced temperature $(T_H - T_L)/T_L$ for $C_H = C_L$ and $T_L < T_H < 3T_L$. Discuss the properties of this function in the limit $T_H \rightarrow T_L$.

Solution: