

[tex103] **Energy fluctuations and thermal response functions**

(a) Show that the following relation holds between the energy fluctuations in the microscopic ensemble and the heat capacity of a system described by a microscopic Hamiltonian  $H$ :

$$\langle (H - \langle H \rangle)^2 \rangle = k_B T^2 C_V.$$

(b) Prove the following relation in a similar manner:

$$\langle (H - \langle H \rangle)^3 \rangle = k_B^2 \left[ T^4 \left( \frac{\partial C_V}{\partial T} \right)_V + 2T^3 C_V \right].$$

(c) Determine the relative fluctuations as measured by the quantities  $\langle (H - \langle H \rangle)^2 \rangle / \langle H \rangle^2$  and  $\langle (H - \langle H \rangle)^3 \rangle / \langle H \rangle^3$  for the classical ideal gas with  $N$  atoms.

**Solution:**