

## [tex110] Occupation number fluctuations

Consider an ideal quantum gas specified by the grand partition function  $Z$ . Start from the expressions,

$$\langle n_k \rangle = -\beta^{-1} \frac{\partial \ln Z}{\partial \epsilon_k} = \frac{1}{z^{-1} e^{\beta \epsilon_k} + a}, \quad \langle n_k^2 \rangle - \langle n_k \rangle^2 = -\beta^{-1} \frac{\partial \langle n_k \rangle}{\partial \epsilon_k},$$

where  $a = +1, 0, -1$  represent the FD, MB, and BE cases, respectively, to derive the following result for the relative fluctuations in the occupation numbers:

$$\frac{\langle n_k^2 \rangle - \langle n_k \rangle^2}{\langle n_k \rangle^2} = \frac{1}{\langle n_k \rangle} - a.$$

Note that in the BE (FD) statistics, these fluctuations are enhanced (suppressed) relative to those in the MB statistics.

**Solution:**