[tex121] Stable white dwarf

Consider a burnt-out white dwarf star. For simplicity we assume that it consists of equal numbers N of electrons, protons, and neutrons. The electrons form a fully degenerate, nonrelativistic Fermi gas that prevents the star from collapsing into a neutron star or a black hole.
(a) Under the assumption that the kinetic energy is predominantly due to the electrons and that the potential energy is predominantly gravitational in nature, show that the total energy of the star depends on N and R (radius) as follows:

$$E = E_{kin} + E_{pot} = \frac{3\hbar^2}{10m_e}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N^{5/3}}{R^2} - \frac{12}{5}m_n^2 G\frac{N^2}{R},$$

where m_e, m_n are the electron and neutron masses, and G is the universal gravitational constant.
(b) Using a star of solar mass, $m_\odot \simeq 1.99 \times 10^{30}$kg, find the radius R_{wd} in units of the solar radius, $R_\odot \simeq 6.96 \times 10^8$m.

Solution: