

[tex158] Some fluid system

Consider a fluid system characterized by the Helmholtz free energy,

$$A(T, V) = -aT^2 \ln \left(\frac{V}{V_0} \right),$$

where a, V_0 are constants.

(a) Infer from this thermodynamic potential the following thermodynamic functions and response functions: (i) the pressure $p(T, V)$, (ii) the entropy $S(T, V)$, (iii) the internal energy $U(T, V)$, and (iv) the heat capacity $C_V(T, V)$. (v) the enthalpy $E(T, p)$, (vi) the heat capacity $C_p(T, p)$.

(b) Determine the functional dependence between pressure and volume during an adiabatic process in the form $f(p, V) = \text{const}$.

(c) Reconstruct the result obtained in (a ii) from integrating the differential

$$dS = \left(\frac{\partial S}{\partial T} \right)_V dT + \left(\frac{\partial S}{\partial V} \right)_T dV,$$

where the two partial derivatives can be extracted from other results obtained in part (a). Identify clearly the path of integration chosen.

Solution: