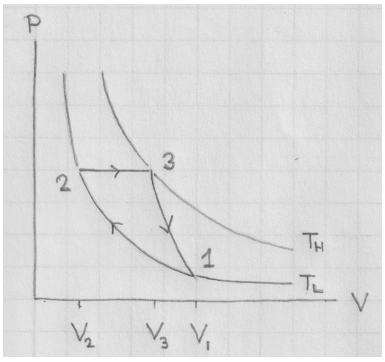
## [tex196] Ideal-gas engine with three-step cycle I

Consider the three-step cycle $1 \to 2 \to 3 \to 1$ shown for 1mol of a monatomic classical ideal gas. Given for this exercise are the relations,

$$pV = RT, \quad C_V = \tfrac{3}{2}R, \quad C_p = C_V + R, \quad U = C_V T, \quad pV^{5/3} = \text{const (adiabate)}.$$

The first step 12 is an isothermal compression at the lower temperature $T_L$, the second step 23 is an isobaric expansion ending at the higher temperature $T_H$, and the third step 31 is an adiabatic expansion back to the lower temperature $T_L$. Each destination has a different volume: $V_2 < V_3 < V_1$.
(a) Express the heat transfer $\Delta Q$, the work performance $\Delta W$, and the change in internal energy $\Delta U = \Delta Q + \Delta W$ for each step. Indicate for each quantity whether it is positive, zero, or negative.
(b) Express the three volume ratios $V_1/V_2$, $V_1/V_3$, and $V_3/V_2$ as functions of $T_H/T_L$.
(c) Express the efficiency of the cycle $\eta \doteq \Delta W_{out}/\Delta Q_{in}$ as a function of $T_H/T_L$.
(d) Find the change of entropy $\Delta S$ in each of the three steps, expressed as functions of $T_H/T_L$.




**Solution:**