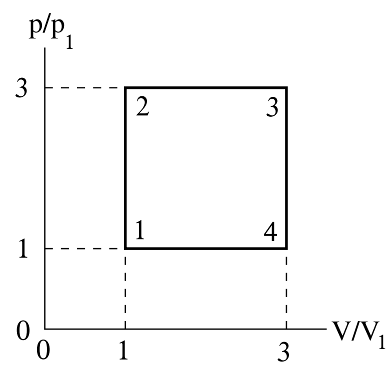
[tex205] Square heat engine with photon gas

Imagine (if you can) a blackbody cavity with controllable temperature T and variable volume V by a piston that can do work. At equilibrium and during quasistatic processes, the photon gas in the cavity has internal energy $U = \sigma T^4 V$ and exerts a pressure $p = \frac{1}{3}\sigma T^4$ on the piston, where σ is the Stefan-Boltzmann constant. During the quasistatic, cyclic process $1 \to 2 \to 3 \to 4 \to 1$ shown the photon gas exchanges heat with the walls and performs work.
(a) Determine the work performance, $\Delta W_{12}, \Delta W_{23}, \Delta W_{34}, \Delta W_{41}$, the change in internal energy, ΔU_{12}, ΔU_{23}, ΔU_{34}, ΔU_{41}, and the heat transfer, $\Delta Q_{12}, \Delta Q_{23}, \Delta Q_{34}, \Delta Q_{41}$, along each step of the cycle. Express these quantities in units of $p_1 V_1$.
(b) Determine the net changes $\Delta W_{net} \doteq \Delta W_{12} + \Delta W_{23} + \Delta W_{34} + \Delta W_{41}$ and analogously for ΔU_{net}, ΔQ_{net} during a complete cycle.
(c) If this cycle is interpreted as a heat engine, its efficiency η is defined as the net work output divided by the total heat absorbed (by the gas). Determine the numerical value of η.

Use the convention for which $\Delta W > 0$ and $\Delta Q > 0$ both contribute to an increase in U.



Solution: