

[tex208] **Canonical partition functions**

Consider two thermodynamic systems, each with N noninteracting degrees of freedom. The canonical partition functions of the two systems are

$$Z_N^{(1)} = \left(1 + \frac{1}{w_1}\right)^N, \quad w_1 = e^{\beta\epsilon}; \quad Z_N^{(2)} = \left(1 + \frac{1}{w_2}\right)^N, \quad w_2 = e^{\beta\epsilon} - 1,$$

where $\beta \doteq 1/k_B T$ and $\epsilon > 0$.

- (a) Determine the Helmholtz free energies $A_1(T, N)$ and $A_2(T, N)$ of the two systems.
- (b) Determine the entropies $S_1(T, N)$ and $S_2(T, N)$.
- (c) Determine the internal energies $U_1(T, N)$ and $U_2(T, N)$.
- (d) Determine the leading asymptotic expressions for U_1 and U_2 at high T .
- (e) Determine the leading asymptotic expressions for S_1 and S_2 at high T .

Solution: