[tex94] Classical ideal gas (grandcanonical ensemble)

Consider a classical ideal gas $[H_N = \sum_{l=1}^{N}(p_l^2/2m)]$ in a box of volume V in equilibrium with heat and particle reservoirs at temperature T and chemical potential μ, respectively.
(a) Show that the grand partition function is $Z = \exp(zV/\lambda_T^3)$, where $z = \exp(\mu/k_BT)$ is the fugacity, and $\lambda_T = \sqrt{h^2/2\pi mk_BT}$ is the thermal wavelength.
(b) Derive from Z the grand potential $\Omega(T,V,\mu)$, the entropy $S(T,V,\mu)$. the pressure $p(T,V,\mu)$, and the average particle number $\langle N\rangle = \mathcal{N}(T,V,\mu)$.
(c) Derive from these expressions the familiar results for the internal energy $U = \frac{3}{2}\mathcal{N}k_BT$, and the ideal gas equation of state $pV = \mathcal{N}k_BT$.

Solution: