

[tex95] Density fluctuations in the grandcanonical ensemble

Consider a system of indistinguishable particles in the grandcanonical ensemble. Derive the following two expressions for the fluctuations in the number of particles N for an open system of volume V in equilibrium with heat and particle reservoirs at temperature T and chemical potential μ , respectively:

$$\langle N^2 \rangle - \langle N \rangle^2 = z \frac{\partial}{\partial z} z \frac{\partial}{\partial z} \ln Z = k_B T V \left(\frac{\partial^2 p}{\partial \mu^2} \right)_{T,V},$$

where $z = \exp(\mu/k_B T)$ is the fugacity, $p(T, V, \mu) = -(\partial \Omega / \partial V)_{T\mu} = -\Omega/V$ is the pressure, and $\Omega(T, V, \mu) = -k_B T \ln Z$ is the grand potential.

Note: Distinguish between the functions $Z(\beta, V, z)$ and $Z(\beta, V, \mu)$. Keep in mind that z depends on μ and T .

Solution: