

[tex98] Ultrarelativistic Bose–Einstein gas

Consider a Bose-Einstein gas with ultrarelativistic one-particle energy $\epsilon_k = c\hbar k = cp$ in the grandcanonical ensemble at temperature T and chemical potential $\mu = 0$.

- (a) Show that the one-particle density of states is $D(\epsilon) = (4\pi V/h^3 c^3)\epsilon^2$.
- (b) Calculate the pressure $p(T)$, the internal energy $U(T, V)$, and the average number of particles in excited states $\mathcal{N}_\epsilon(T, V)$.
- (c) Show that the heat capacity is $C_V/k_B = [16\pi^5/15h^3 c^3]V(k_B T)^3$.

Solution: